

Finalizing Equations Of Motion: Thrust Inputs from Propellers

[latexpage]

This post explains how we determine propeller thrust and drag factors for our quadcopter project.

The last couple of posts have been working out the sum of torque on our quadcopter. A few weeks ago, we covered the gyroscopic effect of the total airframe in the “equations of motion” post.

Next, we looked at the torque induced by the gyroscopic effects of the spinning propellers in the last post.

Now we’ll consider the propeller drive inputs.

Propeller Drive Control Input

We started the, “equations of motion” with the sketch below, so we’ll use it again here as we model the command inputs from the propellers: F_{1-4} .

ref_frames

Torque About Body X-axis

Assume the length of the arm from the center of the quadcopter to the propeller is, ‘l’ then...

$$\tau_x = F_2 \cdot l - F_4 \cdot l$$

The, 'F' terms are thrust forces. We know from an earlier post that this thrust from a propeller is going to be a function of propeller speed squared: ω^2 :

$$F = b \cdot \omega^2$$

Where 'b' is a "thrust factor" we need to determine. After you watch the video below you'll understand how we get...

$$b = \frac{C_t \cdot \rho \cdot D^4}{4\pi^2}$$

where...

C_t is an experimentally determined propeller thrust coefficient, ρ is air density, and D is propeller diameter.

Our torque about the X-axis equation above becomes...

$$\tau_x = b l (\omega_2^2 - \omega_4^2)$$

Torque About Body Y-axis

$$\tau_y = F_1 \cdot l - F_3 \cdot l$$

By the same logic above for the X-axis torque, this Y-axis torque becomes...

$$\tau_y = b l (\omega_1^2 - \omega_3^2)$$

Torque About Body Z-axis

The torque about the Z-Axis is proportional to the squared angular velocities of the propellers. We subtract the squared counter-clockwise props from the clockwise props to lump them. The proportionality constant (the multiplier) is going to be a, "drag factor": d.

See the video for an explanation on how we get 'd' from the experimentally determined, "power coefficient" for a candidate

propeller.

$$\tau_z = d \cdot (\omega_2^2 + \omega_4^2 - \omega_3^2 - \omega_1^2)$$

$$d = \frac{C_p \cdot \rho \cdot D^5}{8\pi^3}$$

C_p is experimentally determined propeller power coefficient. ρ is air density, and D is propeller diameter.

Deriving the, 'b' and, 'd' terms above

'b' is our thrust factor and, 'd' is our drag factor. They are arrived at through experimentally determined thrust and power coefficients for our propeller. We encountered the drag term earlier (here) but we didn't have insight into how it's arrived at.

The University of Illinois and Urbana-Champaign offers an excellent resource: the UIUC Propeller Data Site. It contains wind-tunnel data for a variety of hobby-craft propellers. The following video explains how we could arrive at our drag and thrust factors through measurement.

Propeller Thrust Coefficient

Let's estimate the thrust coefficient from the "static" data because a quadcopter is not going to travel very fast along the propellers' axis. A preliminary estimate for the static thrust coefficient after looking at a number of experimental data sets is...

$$C_t = 0.1$$

From this, we can calculate 'b' as described above.

Propeller Drag Coefficient

Recall the motor+gearbox+propeller post. There we look at how to map the torque load of the propeller back through the gearbox to the motor shaft to get all the parameters correct in modelling the DC motor drive. In that post we have

“Aerodynamic torque for rotor R: $D = 2.91 \times 10^{-7} \frac{Nms^2}{rad^2}$ (page 9, Gafvert) for what looks like a comparable hobby-sized propeller lab set-up”.

We didn't see how this term was derived though.

Confusion...

It's a challenge (for me, at least) to find a propeller with a “drag coefficient” explicitly stated in many references.

Then I read, “While an airfoil can be characterized by relations between *angle of attack*, *lift coefficient*, and *drag coefficient*, a propeller can be described in terms of *advance ratio*, *thrust coefficient*, and *power coefficient*” here. That statement cleared some fog.

Next I found an online question from a person facing the same confusion I was experiencing. The question married the above quote to the same UIUC data I was looking at. The dialog there ultimately gets to a drag term resolved from the UIUC, “power coefficient”.

From there I made sense of it and as expressed in the video above. I derive the ‘d’ term from the experimental “power coefficient,” as explained in the video. As the video explains, I'm estimating...

$$C_p = 0.7$$

For the APC Carbon Fiber 7.8-inch propeller, if we plug in numbers, we calculate ‘d’ as...

$$d = \frac{C_p \cdot \rho \cdot D^5}{8\pi^3} = \frac{(0.7)(1.23 \frac{\text{kg}}{\text{m}^3})(0.175\text{m})^5}{8\pi^3} = 5.65 \times 10^{-7} \text{ Nm s}^2$$

This is close to the number from *page 9 of Gafvert* above. We don't know what specific propeller he is referencing, but we know it is in the family of these hobby propellers. We can be satisfied with this method for arriving at our drag factor from the UIUC data.

Conclusion...

We now have a good understanding of the thrust and drag factors for our propellers.

When we get into the design iterations we might try other propellers with different coefficients. We can revisit the UIUC data and compute a range of 'b' and 'd' terms as we did above. This is how we can assess propeller selection impact on the performance of our quadrotor.